Instructor Guide

Graph Colouring

The activities in this section focused on graph colouring problems.

A vertex colouring of a graph is a way of assigning a colour to each vertex so that two adjacent vertices (i. e. two vertices linked by an edge) never have the same colour. Given a graph, it is always interesting to know the minimal number of colours we need to colour its vertices, this is called the chromatic number of the graph. Vertex colouring has many practical applications (for example, in the context of scheduling, or signal transmission) and is still an active field of research.

You can more about the problem here:

Graph Colouring

The aim of these Activities is for participants to:

- Understand how to simplify real world problems into graph systems by identifying the relevant information in the problem and the irrelevant details that can be discarded.
- Develop logical reasoning and mathematical communication skills.

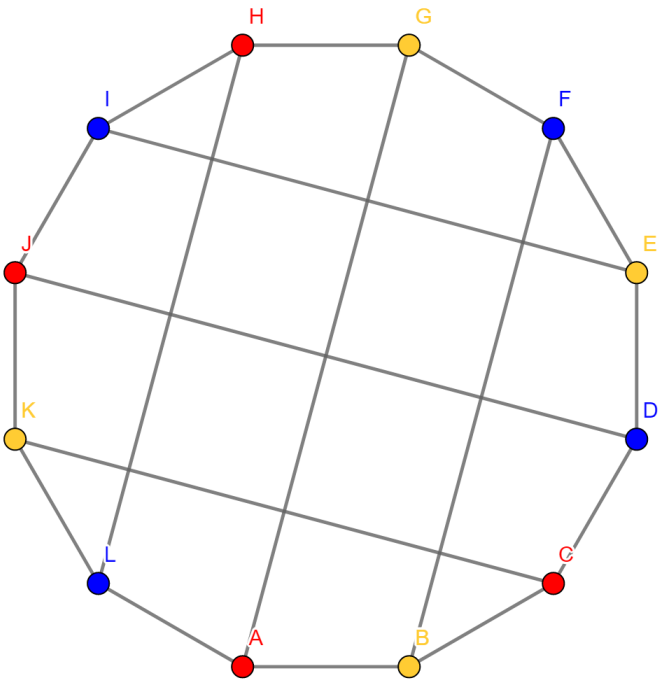
Points to discuss with the students:

- *Is it easier to answer the problems with or without using the graphs?*
- *How do they know that they have found the **minimum** colouring? Can they justify that there isn't a better solution?*

Activity 1: Parties and Radio Problems

Try to get participants to think critically about how to solve the problems as well as how to justify their solutions. Make sure to relate solutions back to original question. You may wish to print multiple copies of the blank graph or laminate the sheet so multiple attempts at colouring can be made.

Party Problem 1 and Radio Problem 1

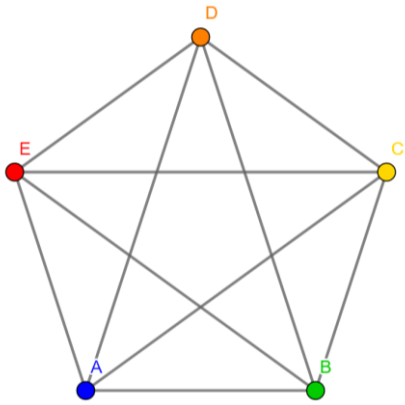


This is not the only possible colouring but it shows that only three colours are required. Two would not be enough:

- Assume we could only use two colours: red and blue
- Let A be red
- Then L and B must be blue
- L is blue so K must be red
- K is red so C must be blue **BUT** B is blue so C must be red.
- C cannot be both red and blue so we must need another colour.

So there must be three different days for the parties to be held on and there must be at least three different radio frequencies used.

Party Problems 2 and 3 and Radio Problem 2

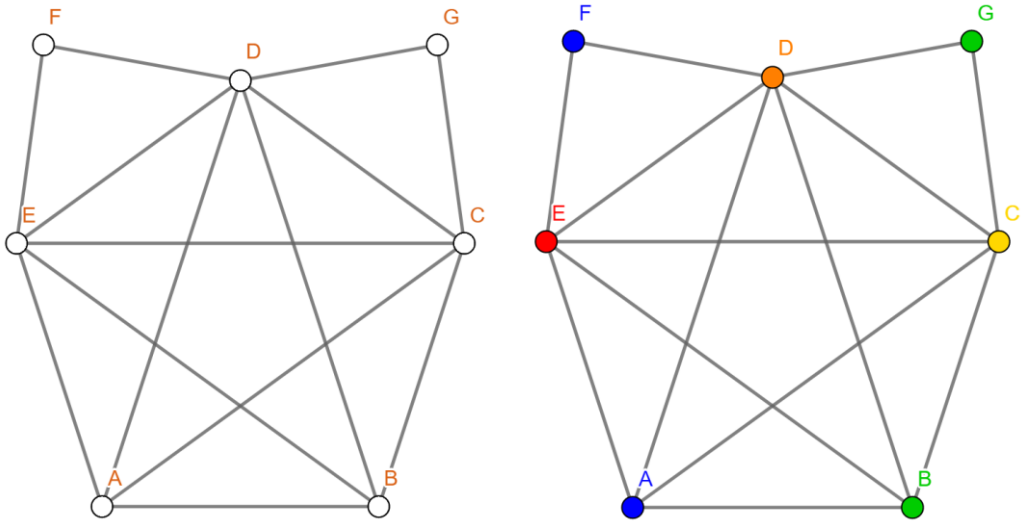


We need five different colours because every vertex is connected to every other one.

So every student must have their party on a different day and five different radio frequencies are required.

This graph is called K_5 because it is the graph with five vertices which is completely connected.

Party Problem 4 and Radio Problem 3



The graph of the system is shown above and can be coloured with 5 colours. By the same reasoning as in Problems 2 and 3 we cannot use less than 5 colours but the extra vertices F and G do not require more colours because they only connect to two other vertices.

So there must be five different days for parties to be held on and five different radio frequencies are required.