

Birthday Party Problem 1

Graph Colouring



In the small town of Mathsyland, there are 12 mathematicians all celebrating their birthday in one week. Those are: Adam, Bella, Charlie, Dan, Ed, Frank, Gustavo, Harry, Ian, Jack, Kim, Lilly.

Unfortunately, they have some friends in common that were invited to two parties at once. For example, Xavier was invited both to Adam's and Bella's party. Below are listed the pairs of people that have friends in common (assume no overlaps):

- Adam – Bella, Adam – Gustavo, Adam – Lilly
- Bella – Charlie, Bella – Frank
- Charlie – Dan, Charlie – Kim
- Dan – Ed, Dan – Jack
- Ed – Frank, Ed – Ian
- Frank – Gustavo
- Gustavo – Harry
- Harry – Ian, Harry – Lilly
- Ian – Jack
- Jack – Kim
- Kim – Lilly

Citizens of Mathsyland do not like noise, so they wish that parties were scheduled in the minimal amount of different time slots.

Taking this into account, how would you schedule the parties so that all the people can go to all the parties they were invited to?

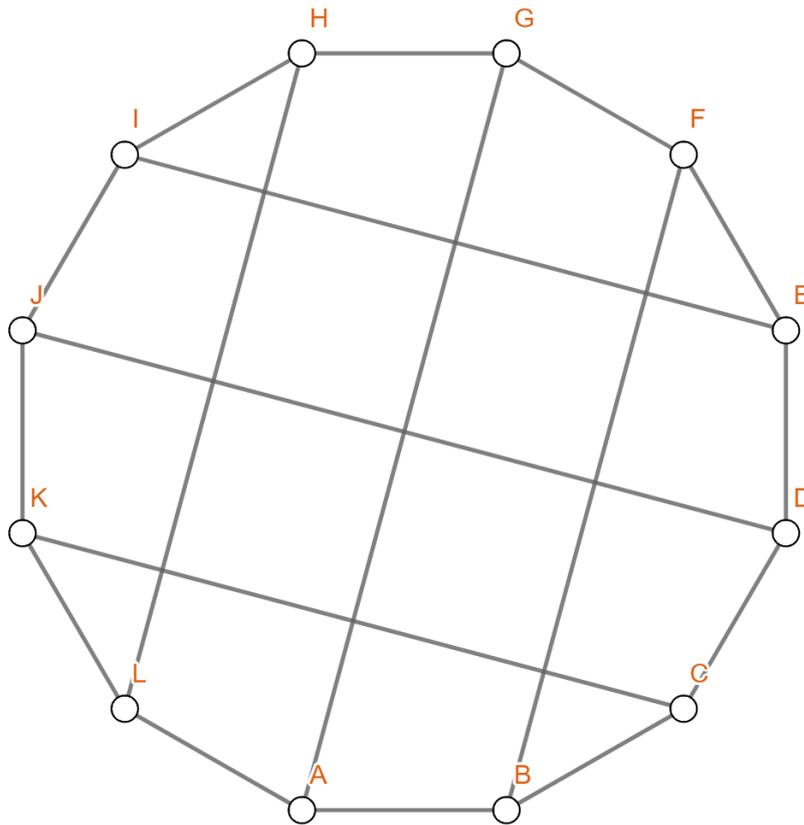
How many time slots did you come up with? Is this the best possible solution? Why? Or, if not, how can it be improved? How can you be certain of having reached the best possible solution?

Turn over the page for a hint at how we can solve this mathematically.

Hint:

The problem of scheduling parties can be simplified into one of colouring a graph.

People celebrating their birthday can be represented as nodes on a graph. That would form, in this case, a 12-node graph. If two people have friends in common, we join the two corresponding nodes by an edge. For instance, there will be an edge between Adam and Bella, another between Adam and Gustavo, since they have friends in common.



Let the colour of a node represent a birthday timeslot. Since the edges represent invited friends in common, essentially we want to colour the nodes in such a way that no edge is pointing to two nodes of the same colour. Furthermore, we want to find minimal amount of different colours needed for this.

Try to colour the graph using the minimal number of colours. How many colours do you need?

Remember, you have to follow this rule: if two nodes are joined by a line then they cannot have the same colour.