

Birthday Party Problem 2

Graph Colouring



In Mathsyland lives a group of students: Alice, Beth, Carl, Dean and Elle, Felix and Gaby. They have known each other since they were born... literally they were all born within the same week! The birthday week is actually approaching, and soon it will be time for celebration.

Since they are a tight group of friends, they all want to participate to each other's birthday party. However, their neighbours are not big fans of parties and want everything to be quiet after a certain time in the night, so the friends have to schedule the parties in the minimal number of time slots. This means that they want to have as many parties not clashing together as possible happening at the same time.

Taking this into account, could you help them scheduling the parties in such a way that everyone can take part in the parties they are invited to?

How many time slots did you come up with? Is this the best possible solution? Why? Or, if not, how can it be improved? How can you be certain of having reached the best possible solution?

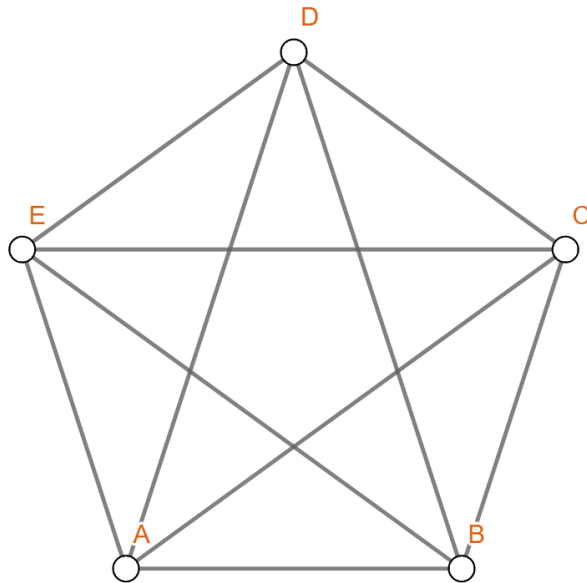
Hint:

The problem of scheduling parties can be simplified into one of colouring a graph.

Consider the people celebrating their birthdays. They can be represented as nodes in a graph: A, B, C, D, and E. To represent the friendship's relationship we draw an edge between two vertices.

Try to construct this graph and turn over to see if you are right.

A graph of this system looks like this:



Let the colour of a node represent a birthday timeslot. Since the edges represent the friendships, we need to colour the nodes in such a way that no edge connects to two nodes of the same colour. Furthermore, we want to find minimal amount of different colours needed for this.

Try to colour the graph using the minimal number of colours. How many colours do you need? How can be sure you've found the minimum number of colours?

Remember, you have to follow this rule: if two nodes are joined by a line then they cannot have the same colour.