Birthday Party Problem 4

Graph Colouring

In Mathsyland lives a group of students: Alice, Beth, Carl, Dean, Elle, Felix and Gaby. They do not know each other very well, but, since the town is very small, they have got many friends in common. Funnily enough, they were all born within the same week!

The birthday week is now approaching, and soon it will be time for celebration. Since they are all very polite, they do not want to put guests in the position where they have to choose between two parties, therefore they meet to compare their guest lists (see below).

Alice: Mary, Simon, Andrew, Charly, Robert

Beth: Mary, Camilla, Frances, Patricia, John

Carl: Simon, Camilla, Frances, Eve, Bianca

Dean: Frances, Andrew, Lucy, Alex

Elle: Lucy, Camilla, Mary

Felix: Patricia, Robert, Amy

Gaby: Eve, John

Unfortunately, the inhabitants of Mathsyland are not big fans of parties and want everything to be quiet after a certain time in the night, so the students have to schedule the parties in the minimal number of time slots. This means that they want to have as many parties not clashing together as possible happening at the same time.

Taking this into account, could you help them scheduling the parties in such a way that everyone can take part in the parties they are invited to?

How many time slots did you come up with? Is this the best possible solution? Why? Or, if not, how can it be improved? How can you be certain of having reached the best possible solution?

Hint:

The problem of scheduling parties can be simplified into one of colouring a graph.

People celebrating their birthday can be represented as nodes on a graph. If two people have friends in common, we join the two corresponding nodes by an edge. For instance, there will be an edge between Alice and Beth, another between Adam and Carl, since they have got friends in common.

Try to construct the graph that represents this system.

Let the colour of a node represent a birthday timeslot. Since the edges represent the friendships in common, we need to colour the nodes in such a way that no edge connects to two nodes of the same colour. Furthermore, we want to find minimal amount of different colours needed for this.

Try to colour the graph using the minimal number of colours. How many colours do you need? How can be sure you've found the minimum number of colours?

Remember, you have to follow this rule: if two nodes are joined by a line then they cannot have the same colour.