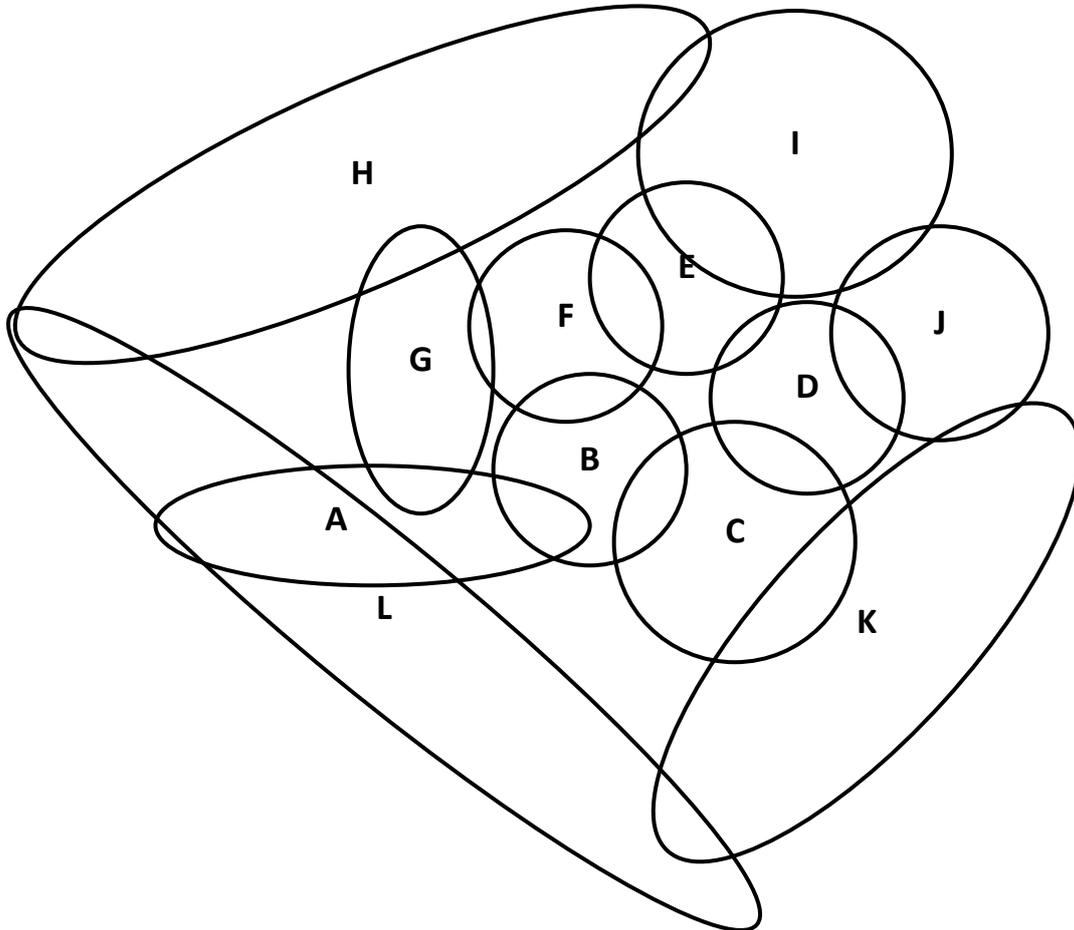


Radio Problem 1

Graph Colouring

In the small town of Radioland, there are 12 radio stations broadcasting mathematical lectures: A, B, C, D, E, F, G, H, I, J, K and L.

Each radio station has a specific area in which people can listen to it, as drawn bellow. Unfortunately, some areas overlap:



The officers of Radioland can license specific wave frequency to each station. But, they want to use the minimal amount of different frequencies, in order to minimize costs. Nevertheless, in an area of overlap, each radio station has to have different frequency; otherwise the listeners would not be able to listen to their station.

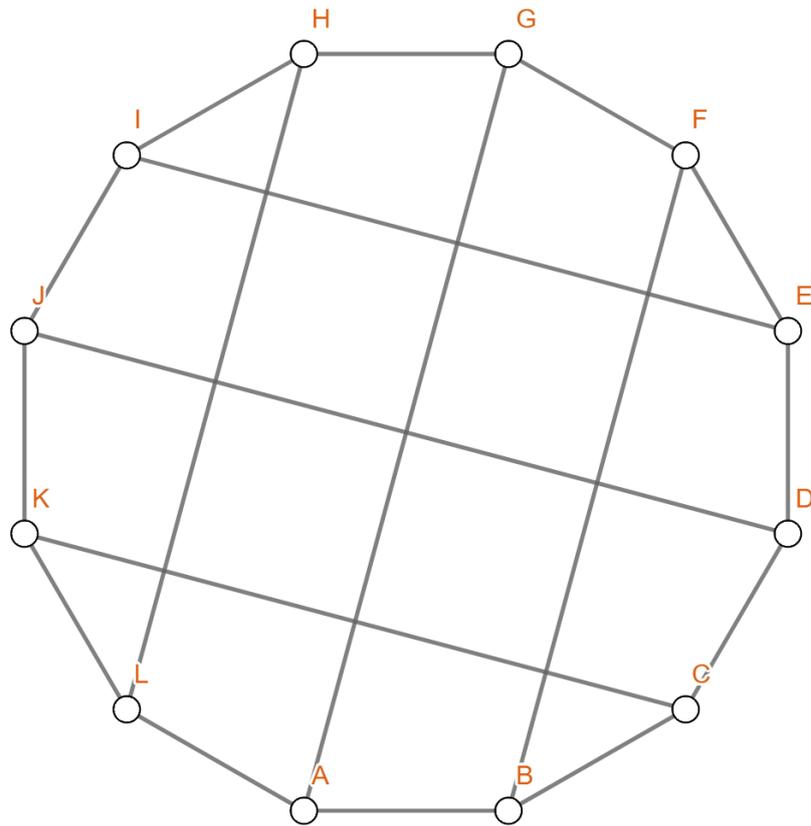
How would you distribute the frequencies amongst the radio stations so that minimal amount of different frequencies is used?

Hint:

The problem of allocating radio frequencies can be simplified into one of colouring a graph. Radio stations can be represented as nodes on a graph. If two radio stations can propagate in areas which overlap, we join the two corresponding nodes by an edge. For instance, there will be an edge between A and B, another between A and G, since their areas overlap.

Try to construct a graph of this system and turn over to see if you are correct.

A graph of this system looks like this:



Let the colour of a node represent a specific radio frequency. Since the edges represent overlapping areas, essentially we want to colour the nodes in such a way that no edge is pointing to two nodes of the same colour. Furthermore, we want to find minimal amount of different colours needed for this.

Try to colour the graph using the minimal number of colours. How many colours do you need?

Remember, you have to follow this rule: if two nodes are joined by a line then they cannot have the same colour.