Instructor Guide

Shannon Switching Game

All the activities in this section are about the Shannon Switching Game – a strategy game for two players invented by Claude Shannon, an American mathematician and electrical engineer.

The game is played on a graph with two special vertices, A and B. Each edge of the graph can be either coloured or removed. The two players are called *Short* and *Cut*, and they take turns to move. On *Cut*'s turn, they delete a non-coloured edge from the graph. On *Short*'s turn, they colour any edge still in the graph. If *Cut* manages to turn the graph into one where A and B are no longer connected, they win. If Short manages to create a coloured path from A to B, they win. The game always terminates after a finite number of moves, and one of the two players has to win.

Either *Short*, *Cut*, or the player moving first is guaranteed the existence of a winning strategy on any given graph:

The aim of these Activities is for participants to:

- Become familiar with the Shannon Switching game
- Develop strategies to win the game as *Cut* or *Short*
- Develop theories about what qualities of the graphs are needed for there to be a winning strategy for a particular player

Note: In all these activities the solutions are not always very obvious. They key for the students is experimentation and to familiarise themselves with the key concepts so don't worry about answering questions with "I don't know"!

Activity 1: Virus Games

This is a set of four Shannon Switching games the "virus" player is trying to connect two points on the map: they are *Short*. The "human" player is trying to stop the virus spreading by cutting links between countries: they are *Cut.*

These problem sheets are A3 size and are designed to be printed and laminated so that participants of the workshop can draw on the sheets with whiteboard pens. Alternatively they could be printed out A4 sized and given to individuals to keep.

Complicated

There is no pattern behind the graph in this worksheet. It is purely to let the students play around with the game and get familiar with the concept. It will be difficult for them to develop a strategy for this game but you should encourage them to start thinking about the properties of the graph when making decisions about their moves rather than acting randomly. For example the "virus" player (*Short*) could look for the quickest route, or the route through countries with lots of connections that will be hard for the "humans" (Cut) to block off. The "human" player might start to look for a bridge in the graph that could easily cut off a whole part of the map for the "virus".

Short Strategy

In this game, the graph can be split into two subgraphs (trees) that connects each vertex.

- *Short* is guaranteed a winning strategy on a graph G if and only if there are two trees, S and T in the graph G, which contain the vertices A and B, and such that S and T contain the same vertices but do not have any edges in common.

- The strategy works as follows: If *Cut* removes an edge from one of the two trees, *Short* finds an edge in the other tree which reconnects the broken tree and colours it.
- If two trees with the above properties cannot be found in G, then either *Cut* has a winning strategy, or the player who makes the first move has a winning strategy.
- A proof of this can be found in this article, but do not feel obliged to read it because it might be a bit of a difficult read. If any question about winning strategies for *Short* comes, you can point the visitors to it.

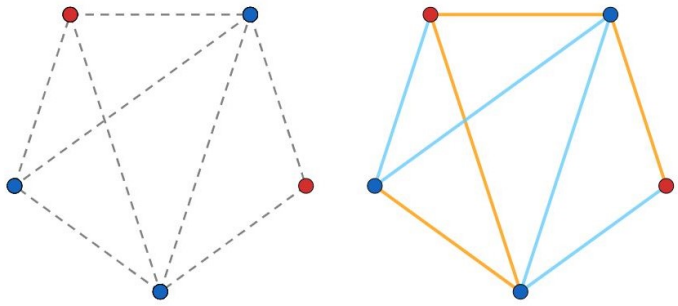
So for this game, the "virus" can always win. Prompt students to play on the second map which shows the two trees in different colours and help them to develop the strategy described above.

4x3 Grid and 8x Grid

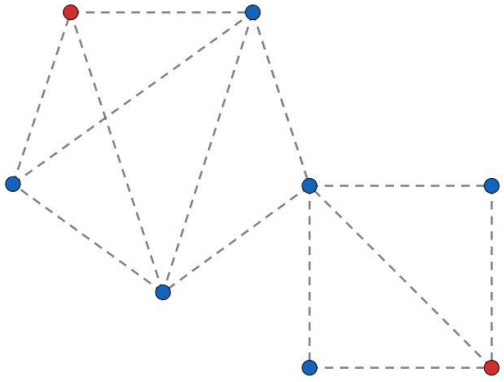
This game is to get the students to start thinking logically about how the structure of the graph affects the playing strategy. Their approach will probably be more random playing on the map compared to when they try playing on the grid, even though the actual structure is the same for both. Ask them to consider why this is (*the simpler layout of the grid helps to see what the important information is in the graph).*

Activity 2: Simple Games

These problem sheets are designed to be printed and laminated so that participants of the workshop can draw on the sheets with whiteboard pens.



Having already played the virus games and been shown the idea of splitting the graphs into trees, students should be on the lookout for this in these simpler games. Encourage them to do this and decide which player these game favour. Game 1 has a short strategy by splitting the graph as shown above. This is not possible for the graph in Game 2. This means that Graph 2 has a strategy for either *Cut* or whichever player goes first.



Activity 3: Design a Graph to Win

This is a simple activity to get students to approach the problem systematically by trying to design a graph that can be won by either *Cut* or *Short.* There are a limited number of graphs that are possible on so few vertices.